Factoring Quadratic Equations

Greatest common factor

1.) $6x^2 - 18x$ the GCF is $6x$

$6x(x-3)$ factor out $6x$

2.) $14xy^3 + 49x^2y^2 + 7xy^2$ the GCF is $7xy^2$

$7xy^2\left(\frac{14xy^3}{7xy^2} + \frac{49x^2y^2}{7xy^2} + \frac{7xy^2}{7xy^2}\right)$ factor out $7xy^2$

$7xy^2(2y + 7x + 1)$

$\frac{x^1}{x^1} = 1$

$x^0 = 1$

Factor by Grouping

There must be four terms

1) $12x^2 + 28x + 15$

$12x^2 + 18x + 10x + 15$ rewrite the center term

$(12x^2 + 18x) + (10x + 15)$ group the 1st 2 terms & the last 2 terms

$6x(2x+3) + 5(2x+3)$ Factor out the GCF. These two must be the same

$(2x+3)(6x+5)$ factor out the common factor

2.) $3x^3 - 7x^2 - 6x + 14$

$(3x^3 - 7x^2) - (6x - 14)$ When a negative is involved it must be factored out when grouping.

$x^2(3x-7) - 2(3x-7)$ Factor out the GCF

$(3x-7)(x^2-2)$ factor out the common factor

Factoring Trinomials

Trinomial	Factored form	Important Observations
$x^2 + 9x + 14 = (x+7)(x+2)$	$7 \& 2$	are factors of 14 whose sum is 9
$y^2 - 8y + 15 = (y-3)(y-5)$	$-3 \& -5$	are factors of 15 whose sum is -8
$x^2 - 2x - 35 = (x-7)(x+5)$	$-7 \& 5$	are factors of -35 whose sum is -2
$y^2 + 4y - 12 = (y+6)(y-2)$	$6 \& -2$	are factors of -12 whose sum is 4

1.) $x^2 + 20x + 75$

$(x+5)(x+15)$

Factors of 75

$1 + 75 = 76$ ✗

$3 + 25 = 28$ ✗

$5 + 15 = 20$ ✓

$ax^2 + bx + c$	both factors are positive
$ax^2 - bx + c$	both factors are negative
$ax^2 + bx - c$	the larger factor is positive, the smaller factor is negative
$ax^2 - bx - c$	the larger factor is negative, the smaller factor is positive

Trinomials with a common factor

1.) $2x^2 - 12x + 16$

$2(x^2 - 6x + 8)$

$2(x-2)(x-4)$

8

$1 \quad 8$

$2 \quad 4$ ✓

both factors are negative

Trinomials with a leading coefficient

1.) $6x^2+19x+10$ factors of 6 factors of 10

1	6
2	3

1	10
2	5

$6\cdot10+1\cdot1=61$
$6\cdot1+1\cdot10=16$
$1\cdot2+6\cdot5=32$
$1\cdot5+6\cdot2=17$
$2\cdot1+3\cdot10=32$
$2\cdot10+3\cdot1=23$
$2\cdot5+3\cdot2=16$
$2\cdot2+3\cdot5=19$ ✓

$6x^2+19x+10=(2x+5)(3x+2)$

Perfect Square Trinomials

ax^2+bx+c

1) $x^2-8x+16$ $\sqrt{16}=4$ $\frac{16}{2}=8$

$(x-4)^2$

When the leading coefficient is 1:
If you can take the square root of c and $\left|\frac{c}{2}\right|=|b|$ then the trinomial is a perfect square

2.) $4x^2+20x+25$ $\sqrt{4}=2$ $\sqrt{25}=5$

$(2x+5)^2$

When the leading coefficient is a square:
You can take the square root of a and c and $2(\sqrt{a})(\sqrt{c})=b$, the trinomial is a perfect square

The Difference of Two Squares

$$a^2 - b^2 = (a+b)(a-b)$$

1.) $x^2 - 49$
$= (\sqrt{x^2} + \sqrt{49})(\sqrt{x^2} - \sqrt{49})$
$= (x+7)(x-7)$

2.) $100 - x^2$
$= (10+x)(10-x)$

3.) $2x^3 - 72x$
$= 2x(x^2 - 36)$
$= 2x(x+6)(x-6)$

4.) $49r^2 - \frac{1}{9}s^2$
$= \left(7r + \frac{1}{3}s\right)\left(7r - \frac{1}{3}s\right)$

Solving Quadratic Equations

Once you have the factors of an equation
Set each factor equal to zero and solve for the variable

- Solving
- Finding the zeros
- Where the function crosses the x-axis

all mean the same thing
Set the equation equal to zero and solve.

The largest exponent determines the maximum number of solutions.

Example	Type of Eq.	Degree	Max # of solutions
$x^2 - x - 6 = 0$	Quadratic	2	2
$2y^3 - 18y = 0$	Cubic	3	3
$a^4 - 5a^2 + 4 = 0$	Quartic	4	4

1) $6x^2 - 18x = 0$

$6x(x-3) = 0$

$\frac{6x}{6} = \frac{0}{6}$ $\quad x - 3 = 0$
$x = 0$ $\quad +3 \; +3$
$x = 3$

$x = \{0, 3\}$ solution set.

2.) $12x^2 + 28x + 15 = 0$

$12x^2 + 18x + 10x + 15 = 0$

$(12x^2 + 18x) + (10x + 15) = 0$

$6x(2x+3) + 5(2x+3) = 0$

$(2x+3)(6x+5) = 0$

$2x+3=0$ $\quad -3 \quad -3$

$\frac{2x}{2} = \frac{-3}{2}$

$x = \frac{-3}{2}$

$6x+5=0$ $\quad -5 \quad -5$

$\frac{6x}{6} = \frac{-5}{6}$

$x = \frac{-5}{6}$

$x = \left\{-\frac{3}{2}, \frac{-5}{6}\right\}$

3.) $x^2 - 8x + 16$

$(x-4)^2$

$x-4=0$

$+4 \quad +4$

$x = 4$

4.) $x^2 - 4 = 0$

$+4 \quad +4$

$\sqrt{x^2} = \pm\sqrt{4}$

$x = \pm 2$

Remember taking the square root across the equal sign is always + and − giving you two solutions.

5.) $2x^3 - 72x = 0$

$= 2x(x^2 - 36) = 0$

$= 2x(x+6)(x-6) = 0$

$2x = 0$ $\quad x+6=0$ $\quad x-6=0$

$x = 0$ $\quad x = -6$ $\quad x = 6$

$x = \{-6, 0, 6\}$